

## **1- Publications in Ship Structural Analysis and Design** **(1969-2002)**

- 1- "Effect of Variation of Ship Section Parameters on Shear Flow Distribution, Maximum Shear Stresses and Shear Carrying Capacity Due to Longitudinal Vertical Shear Forces", European Shipbuilding, Vol. 18. (Norway-1969), Shama, M. A.,
- 2- "Effect of Ship Section Scantlings and Transverse Position of Longitudinal Bulkheads on Shear Stress Distribution and Shear Carrying Capacity of Main Hull Girder", Intern. Shipb. Progress, Vol. 16, No. 184, (Holland-1969), Shama, M. A.,
- 3- "On the Optimization of Shear Carrying Material of Large Tankers", SNAME, J.S.R, March. (USA-1971), Shama, M. A.,
- 4- "An Investigation into Ship Hull Girder Deflection", Bull. of the Faculty of Engineering, Alexandria University, Vol. XII., (Egypt-1972), Shama, M. A.,
- 5- "Effective breadth of Face Plates for Fabricated Sections", Shipp. World & Shipbuilders, August, (UK-1972), Shama, M. A.,
- 6- "Calculation of Sectorial Properties, Shear Centre and Warping Constant of Open Sections", Bull., Of the Faculty of Eng., Alexandria University, Vol. XIII, (Egypt-1974), Shama, M. A.
- 7- "A simplified Procedure for Calculating Torsion Stresses in Container Ships", J. Research and Consultation Centre, AMTA, (EGYPT-1975), Shama, M. A.
- 8- "Structural Capability of Bulk Carriers under Shear Loading", Bull., Of the Faculty of Engineering, Alexandria University, Vol. XIII, (EGYPT-1975), Also, Shipbuilding Symposium, Rostock University, Sept. (Germany-1975), Shama, M. A.,
- 9- "Shear Stresses in Bulk Carriers Due to Shear Loading", J.S.R., SNAME, Sept. (USA-1975) Shama, M. A.,
- 10- "Analysis of Shear Stresses in Bulk Carriers", Computers and Structures, Vol.6. (USA-1976) Shama, M. A.,
- 11- "Stress Analysis and Design of Fabricated Asymmetrical Sections", Schiffstechnik, Sept., (Germany-1976), Shama, M. A.,
- 12- "Flexural Warping Stresses in Asymmetrical Sections" PRADS77, Oct., Tokyo, (Japan-1977), Intern. Conf/ on Practical Design in Shipbuilding, Shama, M. A.,
- 13- "Rationalization of Longitudinal Material of Bulk Carriers, Tehno-Ocean'88, (Jpan-1988), Tokyo, International Symposium, Vol. II, A. F. Omar and M. A. Shama,
- 14- "Wave Forces on Space Frame Structure", AEJ, April, (Egypt-1992), Sharaki, M., Shama, M. A., and Elwani. M.,
- 15- "Response of Space Frame Structures Due to Wave Forces", AEJ, Oct., (Egypt-1992). Sharaki, M., Shama, M. A., and Elwani. M. H.
- 16- "Ultimate Strength and Load carrying Capacity of a Telescopic Crane Boom", AEJ, Vol.41., (Egypt-2002), Shama, M. A. and Abdel-Nasser, Y.

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# Stress Analysis and Design of Fabricated Asymmetrical Sections

M: A. Shama, B. Sc., Ph. D\*

## Summary

The various stress components induced in a uniform fabricated member having an asymmetrical section are examined. Particular emphasis has been placed on the flexural warping stresses induced by torsional loading. Various cases of support conditions are considered. The warping stresses are calculated using the sectorial properties of the section. The effect on warping stresses of support conditions and scantlings of section are examined. A design criterion based on the provision of adequate strength and stability is suggested. A numerical example is given to illustrate the calculation procedure.

It is shown that the proposed method gives results in good agreement with both finite element (FEM) and model test results.

## Introduction:

Fabricated asymmetrical sections are widely used by shipbuilders for longitudinals, girders, ...etc., since they are generally more economical to produce than symmetrical sections. This economy of production may be further improved by using automatic welding (1).

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\* Assoc. Prof., Fac. of Eng., Alexandria University, Egypt.  
Visiting Prof., Fac. of Eng., Basrah University, Iraq.

Horizontal girders, or side stringers, in oil tankers and bulk carriers are normally made asymmetrical so as to reduce the amount of sludge, or grains,...etc., accumulated over the top surface of the web plate.

Asymmetrical sections, however, are not structurally efficient (2,3,4), because of the additional flexural warping stresses induced by torsional loading. The latter results from the offset position of the shear centre associated with asymmetrical sections.

This paper examines the various stress components induced in uniform members having asymmetrical sections. The emphasis is placed on the flexural warping stress, which is calculated using the sectorial properties of the section (5). Uniform and linear load distributions are both considered, together with various support conditions. The effects of support conditions and scantlings of section on the magnitude of the flexural warping stress are examined. A numerical example is given for this purpose. The necessary conditions to ensure sufficient strength and stability are suggested.

It is shown that the proposed procedure gives results in good agreement with both finite element method, FEM, and model test results.

#### 1. Flexural stresses in uniform members having asymmetrical sections.

The total flexural stress, at any point, in a uniform member, such as a longitudinal having an asymmetrical section is given by :

$$\sigma_T = \sigma_H + \sigma_b + \sigma_\omega \quad (1.1)$$

where :

- $\sigma_T$  = total flexural stress,
- $\sigma_H$  = hull girder bending stress,
- $\sigma_b$  = flexural stress induced by local loading,
- $\sigma_\omega$  = flexural warping stress.

The design of these longitudinal members is based on the following condition :

$$\sigma_T \leq \text{allowable stress} \quad (1.2)$$

In order to satisfy this condition, each component of  $\sigma_T$  must be made as low as possible.

### 1.1 - Hull girder stress, $\sigma_H$

Hull girder stress,  $\sigma_H$ , depends on the magnitude of hull bending moment, flexural properties of ship section under consideration and on the location of the member relative to the neutral axis of ship section. The mean stress over the asymmetrical section, see fig.(1), is given by :

$$\sigma_H = \frac{M_\xi \cdot \eta}{I_\xi} - \frac{M_\eta \cdot \xi}{I_\eta} \quad (1.3)$$

where :  $M_\xi$  and  $M_\eta$  = hull girder bending moments about  $\xi$  and  $\eta$  axes respectively,

$I_\xi$  and  $I_\eta$  = second moment of area of ship section about  $\xi$  and  $\eta$  axes respectively,

$\xi$  and  $\eta$  = centroidal axes of ship section.

### 1.2 - Local bending stress, $\sigma_b$

The local stress,  $\sigma_b$ , depends on the flexural properties of section, intensity of local loading and the degree of constraint at both ends of member. It varies linearly over the section, see fig.(2) and could be calculated using the simple beam theory.

For the particular case of an isolated uniform member loaded in the (X - Z) plane, by a linearly varying load, the bending moments at both ends are given by :

$$(M_y)_0 = - \frac{f_o L^2}{60} \cdot \left\{ q_L (7 - 3f_L) / 2 + q_o (4 - f_L) \right\} \quad (1.4)$$

$$(M_y)_L = \frac{f_L L^2}{60} \cdot \left\{ q_o (7 - 3f_o) / 2 + q_L (4 - f_o) \right\} \quad (1.5)$$

For the special case when  $q_o = q_L = q$ , the bending moments at both ends, see fig.(3), are given by :

$$(M_y)_0 = - \frac{qL^2}{12} (3 - f_L) \cdot f_o / 2 \quad (1.6)$$

$$(M_y)_L = \frac{qL^2}{12} (3 - f_o) \cdot f_L / 2 \quad (1.7)$$

where :  $q$  = uniform load, tons/m,

$(M_y)_i, (i = 0, L)$  = bending moment at  $x = i, (i = 0, L),$

$f_i, (i = 0, L) =$  degree of constraint at  $x = i, (i = 0, L)$ , such that :

$$0 \leq f_i \leq 1.0, \quad i = 0, L \quad (1.8)$$

## 2. Flexural warping stress, $\sigma_\omega$

The flexural warping stresses induced in a uniform member having an open section are calculated as follows (6) :

$$\sigma_\omega = - E \cdot \phi'' \cdot \omega \quad (2.1)$$

where :  $E =$  modulus of elasticity,

$\phi =$  angle of twist,

$$\phi'' = \frac{d^2 \phi}{dx^2},$$

$\omega =$  principal sectorial coordinate (5).

Thus, for the outer and inner points of the face plate,  $\sigma_\omega$  is given by :

$$(\sigma_\omega)_j = - E \cdot \phi'' \cdot \omega_j, \quad j = o, i \quad (2.2)$$

where  $o$  and  $i$  stand for outer and inner respectively.

It is evident that  $\sigma_\omega$  depends on the principal sectorial properties of section and  $\phi''$ . The former is calculated as given by Shama (5) and the latter is determined from the solution of the torsion equation.

$$C \cdot \phi' - C_1 \cdot \phi''' = T_x \quad (2.3)$$

where :  $C$  and  $C_1$  are the torsional and warping rigidities respectively.

The torsional loading,  $T_x$ , results from the presence of the shear centre,  $C'$ , on the opposite side of the face plate (7), see fig.(4), at a distance  $e$  from the resultant of the lateral pressure. However, if the member is part of a stiffened panel, the shear centre should lie within the outer plating, as the latter cannot deform in its own plane. The modes of deformation of a stiffened panel having symmetrical and asymmetrical sections are shown in fig.(5). It is evident that asymmetrical stiffeners rotate about enforced centres of rotation.

Thus, for uniform loading, the torsional moment is given by :

and for the linear load distribution, the torsional moment is given by :

$$T_x = M_0 + \frac{qex^2}{2L} \quad (2.5)$$

These two loading cases are used to solve equation (2.3), see Appendix (1), for different support conditions. A summary of the solutions is given in tables (1) and (2).

Since  $\omega$  varies linearly over each element of the section,  $\sigma_\omega$  also varies linearly, see fig.(6). It is evident from fig.(6) that the face plate of an asymmetrical section is deficient in comparison with symmetrical sections.

This deficiency of asymmetrical face plates could be easily illustrated by considering a long narrow plate loaded by shear forces along one edge while the other edge is free, see fig.(7). The normal stress over the section is given by :

$$\sigma_n = N / b.t \quad (2.6)$$

and the bending stress is given by :

$$\sigma_b = 3N / b.t \quad (2.7)$$

The maximum stress at the loaded edge is therefore given by :

$$(\sigma_t)_i = 4N / b.t \quad (2.8)$$

and the stress at the free edge is given by :

$$(\sigma_t)_o = - 2N / b.t = - \frac{1}{2}(\sigma_t)_i \quad (2.9)$$

These results indicate clearly the deficiency of asymmetrical face plates.

### 3. Variation of $\sigma_\omega$ with support conditions and scantlings of section.

In order to investigate the effects of support conditions and scantlings of section on the magnitude of  $\sigma_\omega$ , a uniform member having the particulars given in table (3) is considered. The section parameters investigated are the thickness and width of face plate and thickness of web plate. Different cases of support conditions are also considered.

### 3.1 - Effect of scantlings of section on $\sigma_\omega$

The effect of variation of  $b$ ,  $t_f$ ,  $t_w$  on  $\sigma_\omega$  is given in table (4). The total local stress, ( $\sigma_t = \sigma_b + \sigma_\omega$ ), is also given in table (4) for the inner edge, where  $\sigma_t$  attains its maximum value.

It is evident from table (4) that increasing web thickness has a significant effect on  $\sigma_\omega$ , whereas increasing the thickness of face plate has a much less effect. Also, increasing the width of face plate has a negligible effect on  $\sigma_\omega$ .

### 3.2 - Effect of support conditions on $\sigma_\omega$

The influence of support conditions for the case of uniform loading is shown in table (5). The results of the unconstrained warping condition is given only for the sake of comparison, as this support condition is not common in practice.

It is clear from table (5) that the degree of constraint at both ends of a member have a marked influence on the magnitude of  $\sigma_\omega$ .

## 4. Comparison with FEM and model test results.

In order to confirm the validity of the proposed approach, a comparison is made with results obtained from analyses by FEM and model testing.

### a. Stress analysis using FEM.

The stresses and deformations of two structural models having asymmetrical sections are examined (8) :

#### i - Structural model I

This model is composed of a uniform member fixed at both ends and subjected to uniform lateral loading. The 3 - D structural idealization is shown in fig.(8). The distribution of the combined bending and flexural warping stresses, along the length of member, is shown in fig.(9), for the inner and outer edges of the face plate. The bending stress, calculated according to the simple beam theory, is also shown in fig.(9). The deformed shape of the section is shown in fig.(10), for two values of the web thickness.

It is evident that increasing web thickness reduces torsional effects.

The deficiency of the asymmetrical face plate could be easily seen in fig.(11), where the results of the 3 - D stress



analysis are compared with the results of the 2 - D stress analysis. In the 2 - D structural idealization, the face plate is replaced by bar elements having sectional area varying from 10% to 100% of the original area of face plate. It is obvious that the efficiency of asymmetrical face plates is not more than 30%.

ii - Structural model II

This model is composed of three members so as to obtain more realistic boundary conditions. The deformed shape of the model is shown in fig.(12). The deficiency of the asymmetrical face plate could be obtained from the results of the 2 - D and 3 - D stress analyses, as shown in fig.(13).

Table (6) compares FEM results with corresponding values calculated according to the proposed approach and the simple beam theory.

b. Stress analysis using model testing

The results of a test model composed of five members having asymmetrical sections, see fig.(14), are obtained from reference (9). The measured total stress at the inner and outer edges of the face plate, at the fixed end, is given in table (6) together with the corresponding calculated values using the proposed method and the simple beam theory.

It is evident from these results that the simple beam theory can not be used to predict the flexural stresses in asymmetrical sections. The comparison with FEM and model test results indicates that the proposed procedure could be satisfactorily used in the stress analysis and design of asymmetrical sections. It is also evident that asymmetrical sections are deficient in carrying lateral loading because of the high stresses developed at the junction between the web and face plate. The twist of the section clearly indicates the effect of torsional loading induced by the lateral pressure.

5. Design criterion

The design of an asymmetrical section could be based on the following conditions :

$$\sigma_T \leq \sigma_i \quad , \quad i = y, c \quad (5.1)$$

where :  $\sigma_i$  , (i = y, c) = yield stress and critical buckling stress respectively.

$$\sigma_c = K \cdot \frac{\pi^2 E}{12(1 - \nu^2)} \cdot \left(\frac{t_f}{b}\right)^2 \quad (5.2)$$

The constant K depends on the boundary conditions of the plate. For the face plate of an asymmetrical section,  $K = 0.43$ , as obtained from reference (10).

For a bottom longitudinal of a ship, conditions (5.1) become :

$$(\sigma_b + \sigma_\omega) \leq (\sigma_i - \sigma_H), \quad i = y, c \quad (5.3)$$

The hull girder stress,  $\sigma_H$ , is the summation of the stillwater and wave induced stress components (11). The maximum allowable value is assigned by Classification Societies (12) and is approximately  $1.6 \text{ t/cm}^2$ . For shipbuilding steel,  $\sigma_y$  is approximately  $2.4 \text{ t/cm}^2$ . Therefore, conditions (5.3) are given by :

$$(\sigma_b + \sigma_\omega) \leq 0.8 \quad (5.4)$$

$$\text{and } (\sigma_b + \sigma_\omega) \leq (800 \left(\frac{t_f}{b}\right)^2 - 1.6) \quad (5.5)$$

It should be noted that expression (5.5) is valid only when :

$$\sigma_c \leq \sigma_y / 2 \quad (5.6)$$

If this condition is not satisfied, the Johnson Ostenfeld hypothesis (13) is used, i.e.

$$(\sigma_c)_m = \sigma_y (1 - \sigma_y / 4 \sigma_c) \quad (5.7)$$

where :  $(\sigma_c)_m$  = modified critical buckling stress.

## 6. Concluding remarks.

From the foregoing analyses, it is concluded that :

- i- Asymmetrical fabricated sections may be subjected to high flexural stresses as a result of the indirect torsional loading induced by the lateral pressure.
- ii- The induced flexural warping stresses may attain values much higher than those calculated using the simple beam theory.
- iii- Web thickness and the degree of constraint at both ends of member have a marked influence on the magnitude of flexural warping stresses.
- iv- Narrow thick face plates having adequate stability are much more efficient than thin broad plates whose effectiveness does not exceed 30%.

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## Appendix (1)

### Solution of the torsion equation

The general equation of a uniform member under non-uniform torsional loading is given by (14) :

$$C \cdot \phi' - C_1 \cdot \phi''' = T_x \quad (\text{A.1})$$

where :  $C = G \cdot J_t$  ,  $J_t =$  torsion constant.

$C_1 = E \cdot J(\omega)$  ,  $J(\omega) =$  warping constant.

The solution of equation (A.1) depends on the mathematical representation of the torque  $T_x$ . Consider the following two cases :

#### a. Uniform loading.

The solution of equation (A.1) for a uniform torque is given by :

$$\phi = \frac{\phi'_0}{k} \cdot \sinh kx + \frac{\phi''_0}{k^2} \cdot (\cosh kx - 1) + \frac{M_0}{C} \left( x - \frac{\sinh kx}{k} \right) + \frac{qex^2}{2C} \quad (\text{A.2})$$

where ;  $T_x = M_0 + qex$  ,  $M_0 =$  end torque. (A.3)

$$k = \sqrt{C / J_1}$$

$\phi_0$ ,  $\phi'_0$  and  $\phi''_0$  are arbitrary constants of integration.

#### b. Linear loading.

In this case, the torque distribution is given by :

$$T_x = M_0 + qex^2 / 2L \quad (\text{A.4})$$

The solution of equation (A.1) is the sum of the complementary function and the particular integral. The former is given by :

$$\phi = A_0 + A_1 \cdot \cosh kx + A_2 \cdot \sinh kx \quad (\text{A.5})$$

and the particular integral is assumed to take the form :

$$\phi = \sum_{i=0}^5 a_i \cdot x^i \quad (\text{A.6})$$

Therefore, the total solution of equation (A.1) is given by :

$$\phi = A_0 + A_1 \cdot \cosh kx + A_2 \cdot \sinh kx + \left( \frac{qe}{L^2 C k^2} + \frac{M_0}{C} \right) \cdot x + \frac{qex^3}{6LC} \quad (\text{A.7})$$

where :  $A_i$  ,  $(i = 0, 1, 2)$  are arbitrary constants to be determined from the end conditions.

The solution of equations (A.2) and (A.7) are obtained for different support conditions. A summary of these solutions is given in tables (1) and (2), see fig.(15).

TABLE (1)

CASE	SUPPORT CONDITION	$\phi$ and $\phi''$ FOR UNIFORM LOADING
1	At $x=0$ and $x=L$ $\phi = \phi' = 0$	$\phi = \frac{qeL}{kC} \left[ \frac{\sinh \frac{kx}{2} \cdot \sinh \frac{k(L-x)}{2}}{\sinh \frac{kL}{2}} - \frac{kx(L-x)}{2L} \right]$ $\phi'' = \frac{qeL}{2C} \left[ \frac{2}{L} - \frac{k \cdot \cosh(kx - \frac{kL}{2})}{\sinh \frac{kL}{2}} \right], \quad \phi''_{\max} = (\phi'')_{x=0} = (\phi'')_{x=L}$
2	At $x=0$ and $x=L$ $\phi = \phi'' = 0$	$\phi = \frac{qe}{C} \left[ \frac{\cosh kL - 1}{k^2 \cdot \sinh kL} \right] \cdot \sinh kx - \frac{1}{k^2} (\cosh kL - 1) - \frac{xL}{2} + x^2$ $\phi'' = \frac{qe}{C} \left[ \frac{\cosh kL - 1}{\sinh kL} \right] \cdot \sinh kx - (\cosh kx - 1), \quad \phi''_{\max} = (\phi'')_{x = \frac{L}{2}}$
3	At $x=0$ $\phi = \phi' = 0$ and at $x=L$ $\phi = \phi'' = 0$	$\phi = \frac{qe}{2C} \left[ - \left\{ \frac{(kL^2 - 2) \cdot \sinh kL + 2kL}{k^2 (kL \cdot \cosh kL - \sinh kL)} \right\} (\cosh kx - 1) - \left\{ \frac{(kL^2 - 2) \cdot \cosh kL + 2}{k(kL \cdot \cosh kL - \sinh kL)} \right\} \left( x - \frac{\sinh kx}{k} \right) + x^2 \right]$ $\phi'' = \frac{qe}{C} \left[ - \left\{ \frac{(kL^2 - 2) \cdot \sinh kL + 2kL}{2(kL \cdot \cosh kL - \sinh kL)} \right\} \cdot \cosh kx + \left\{ \frac{(kL^2 - 2) \cdot \cosh kL + 2}{2(kL \cdot \cosh kL - \sinh kL)} \right\} \cdot \sinh kx + 1 \right]$ $(\phi'')_{x=0} = - \frac{qeL}{C} \left[ \frac{2(\cosh kL - 1) - kL \cdot \sinh kL}{2(\cosh kL - \sinh kL)} \right]$ $\phi''_{\max} = (\phi'')_{x=x_m}, \quad \text{where } x_m = \frac{1}{k} \tanh^{-1} \left[ \frac{(kL^2 - 2) \cdot \cosh kL + 2}{(kL^2 - 2) \cdot \sinh kL + 2kL} \right]$

TABLE (2)

CASE	SUPPORT CONDITION	$\phi$ and $\phi''$ FOR LINEAR LOADING
1	At $x=0$ and $x=L$ $\phi = \phi' = 0$	$\phi = \frac{qe}{6C} \left[ \left( \frac{3}{\sinh kL} - \beta \cdot \tanh \frac{kL}{2} \right) (1 - \cosh kx) + (kx - \sinh kx) \cdot \beta + \frac{kx^3}{L^2} \right], \beta = \frac{3(\cosh kL - 1) - kL \cdot \sinh kL}{kL \cdot \sinh kL + 2(1 - \cosh kL)}$ $\phi' = -\frac{qeL}{6C} \left[ \frac{\beta}{\sinh kL} - \beta \cdot \tanh \frac{kL}{2} \right] \cdot k \cosh kx + k \cdot \beta \cdot \sinh kx - \frac{3kx^2}{L^2}$ $(\phi'')_{x=0} = -\frac{qeL}{6C} \left[ \beta \cdot \tanh \frac{kL}{2} - \frac{3}{\sinh kL} \right], (\phi'')_{x=L} = -\frac{qeL}{6C} \left[ \beta \tanh \frac{kL}{2} - \frac{3}{\sinh kL} \right] \cosh kL - \beta \sinh kL + \frac{6}{kL}$
2	At $x=0$ and $x=L$ $\phi = \phi' = 0$	$\phi = -\frac{qe}{C} \left[ \frac{\sinh kx}{k^2 \sinh kL} + \left( \frac{1}{k^2 L} - \frac{1}{6} \right) \cdot x + \frac{x^3}{6L} \right]$ $\phi'' = -\frac{qe}{C} \left[ \frac{\sinh kx}{\sinh kL} - \frac{x}{L} \right], \phi''_{\max} = (\phi'')_{x=x_m} = \frac{1}{k} \cosh^{-1} \frac{\sinh kL}{kL}$
3	At $x=0$ $\phi = \phi' = 0$ and at $x=L$ $\phi = \phi' = 0$	$\phi'' = \left[ \frac{qe}{Ck^2 \cosh kL} - \left( \frac{T_0}{kC} + \frac{qe}{LCK^3} \right) \cdot \tanh kL \right] (1 - \cosh kx) - \left( \frac{T_0}{Ck} + \frac{qe}{LCK^3} \right) (\sinh kx - kx) + \frac{qe x^3}{6CL}$ <p>where, <math>T_0 = \frac{qe}{k} \left[ \frac{kL - \sinh kL - \frac{6}{kL} \cdot \cosh kL}{kL(kL - \tanh kL) \cdot \cosh kL} \right]</math></p> $\phi'' = -k^2 \left[ \frac{qe}{Ck^2 \cosh kL} - \left( \frac{T_0}{Ck} + \frac{qe}{LCK^3} \right) \cdot \tanh kL \right] \cdot \cosh kx - k \left( \frac{T_0}{C} + \frac{qe}{LCK^2} \right) \cdot \sinh kx + \frac{qe x}{CL}$ $(\phi'')_{x=0} = -\left( \frac{kT_0}{C} + \frac{qe}{kLC} \right) \cdot \tanh kL + \frac{qe}{C \cdot \cosh kL}$

TABLE (3)

Scantlings of Section

Item	Scantlings
length , L	520 cm
face plate , b x t <sub>f</sub>	250 x 25 mm
web plate , d x t <sub>w</sub>	950 x 20 mm
outer plate , S x t <sub>p</sub>	1155 x 27.5 mm

TABLE (4)

Effect of Scantlings of Section on  $\sigma_\omega$

Stress	20			30		
	b = 250		b = 315	b = 250		b = 315
	t <sub>f</sub> = 25	t <sub>f</sub> = 30	t <sub>f</sub> = 30	t <sub>f</sub> = 25	t <sub>f</sub> = 30	t <sub>f</sub> = 30
( $\sigma_\omega$ ) <sub>i</sub> /q	1.082	1.125	1.120	0.608	0.650	0.683
( $\sigma_\omega$ ) <sub>c</sub> /q	-3.615	-3.375	-3.305	-2.865	-2.720	-2.505
( $\sigma_b$ ) <sub>1</sub> /q	2.085	1.893	1.842	1.740	1.608	1.437
( $\sigma_t$ ) <sub>1</sub> /q	3.167	3.018	2.962	2.348	2.258	2.120

TABLE (5)

Effect of Support Conditions on  $\sigma_\omega$

Case	Position	( $\sigma_\omega$ ) <sub>i</sub> / q	( $\sigma_\omega$ ) <sub>c</sub> / q
1	x = 0 or L	1.082	- 3.615
2	x = L/2	4.620	-15.460
3	x = 0	5.160	-17.250
4	x = x <sub>m</sub>	0.895	- 2.990

TABLE (6)

Correlation of Results of FEM, Model Testing,  
Proposed Method and Simple Beam Theory

Type of analysis	$(\sigma_t)_i$ , t/cm <sup>2</sup>	$(\sigma_t)_o$ , t/cm <sup>2</sup>
<u>Model I</u>		
FEM	0.648	- 0.296
Proposed method	0.695	- 0.283
Simple beam theory	0.334	0.334
<u>Model II</u>		
FEM	1.076	- 0.305
Proposed method	0.952	- 0.310
Simple beam theory	0.530	0.530
<u>Test Model</u>		
Test results	476.0 lb/in <sup>2</sup>	- 244 lb/in <sup>2</sup>
Proposed method	478.1 "	- 286.7 "
Simple beam theory	239.5 "	239.5 "



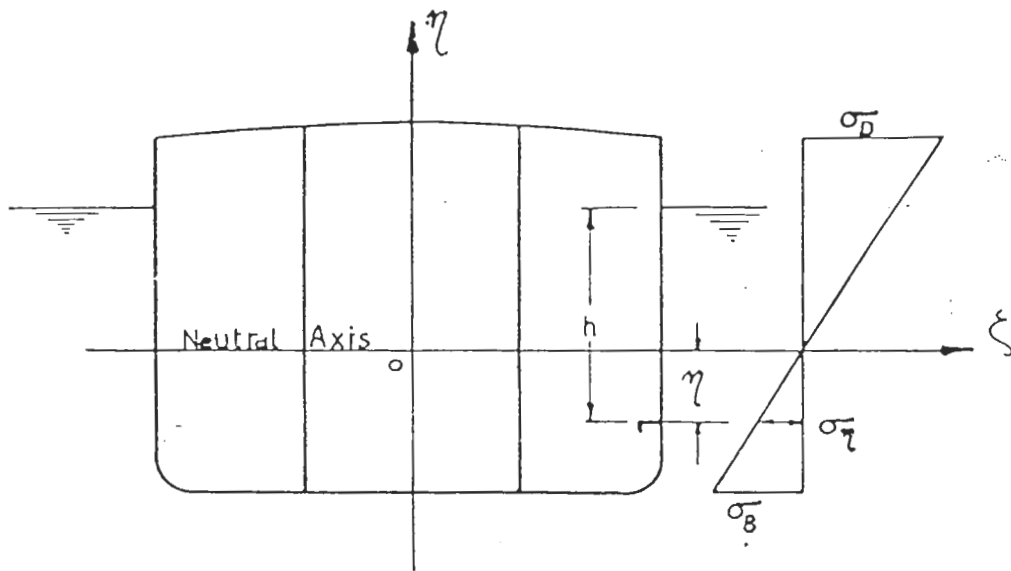


Fig.( 1 ), Hull Girder Bending Stress .

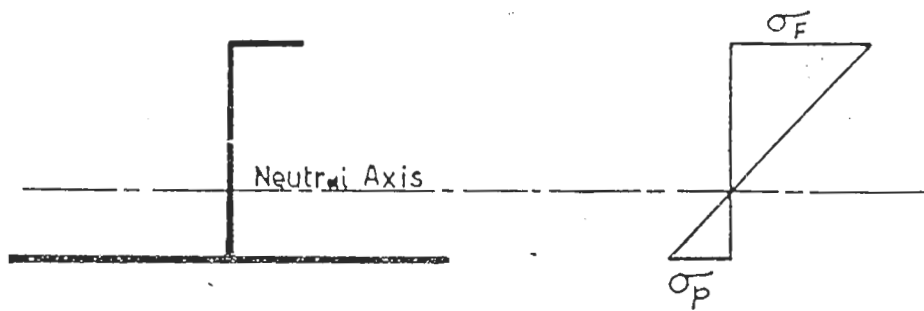
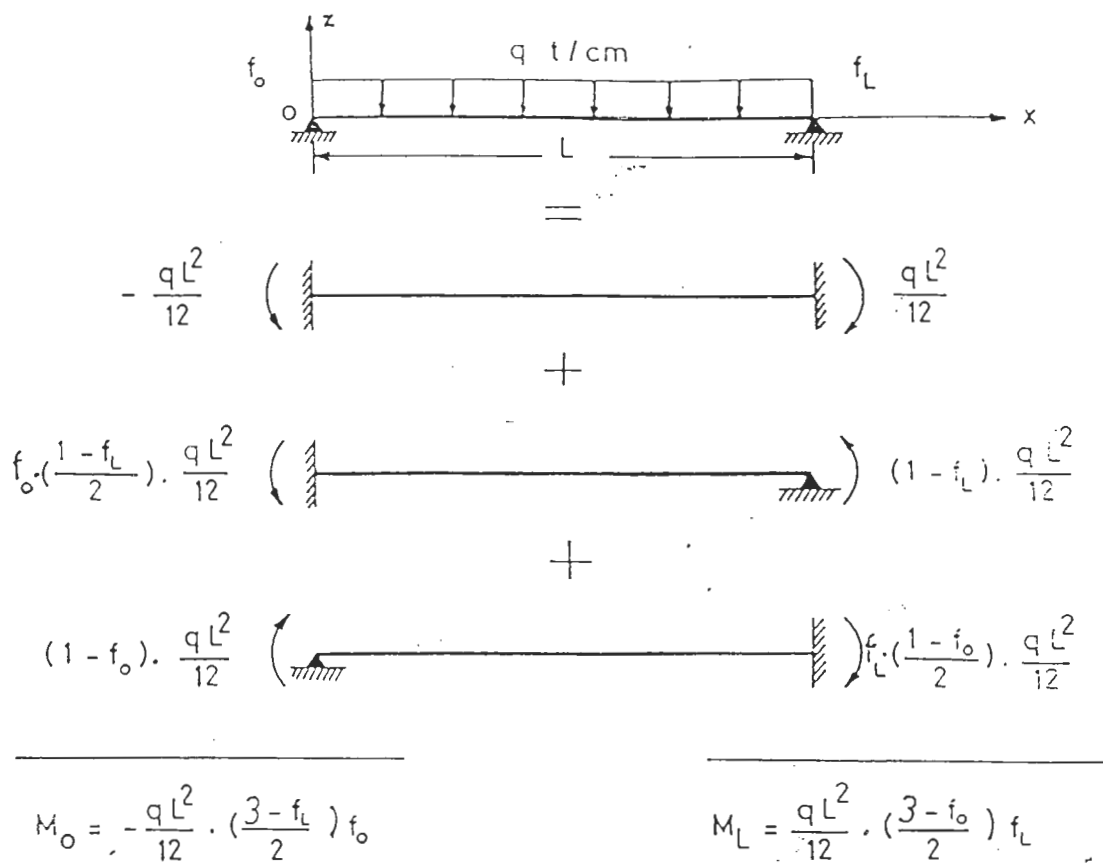


Fig.( 2 ),  $\sigma_b$  Distribution.



- $M_0$  = bending moment at  $x = 0$
- $M_L$  = bending moment at  $x = L$
- $f_0$  = degree of fixity at  $x = 0$
- $f_L$  = degree of fixity at  $x = L$

FIG. (3), END MOMENTS FOR A CONSTRAINED MEMBER

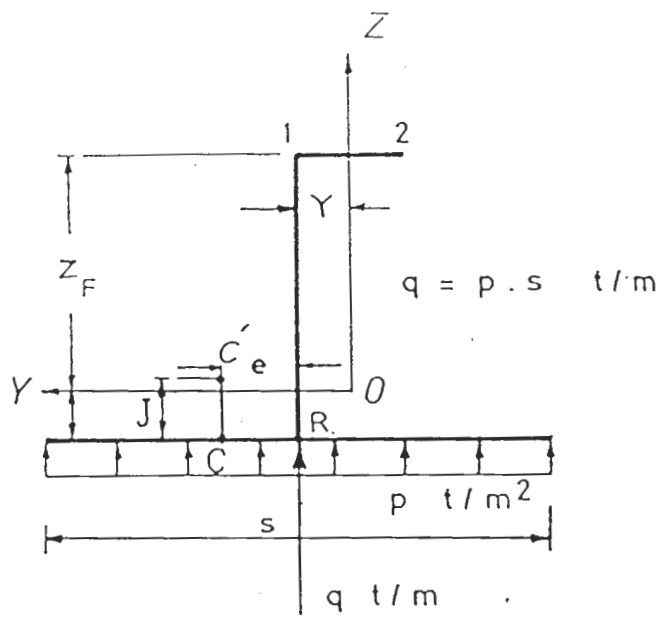
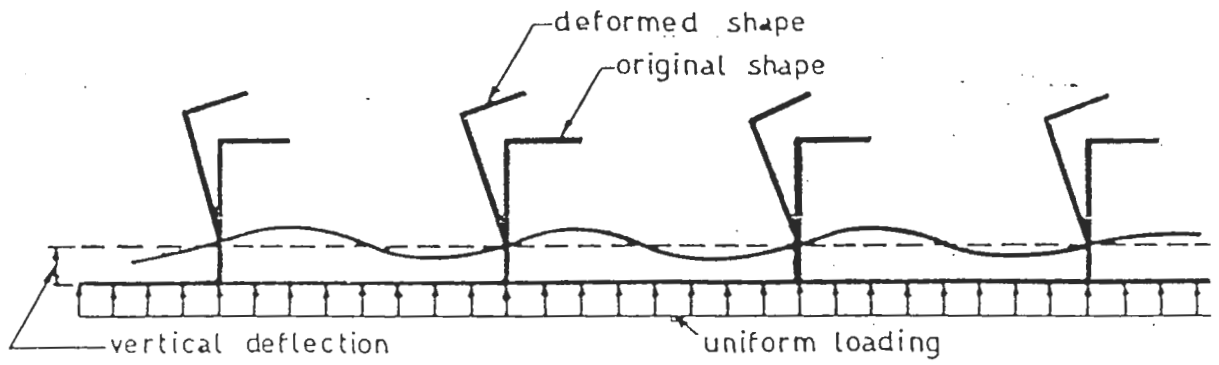
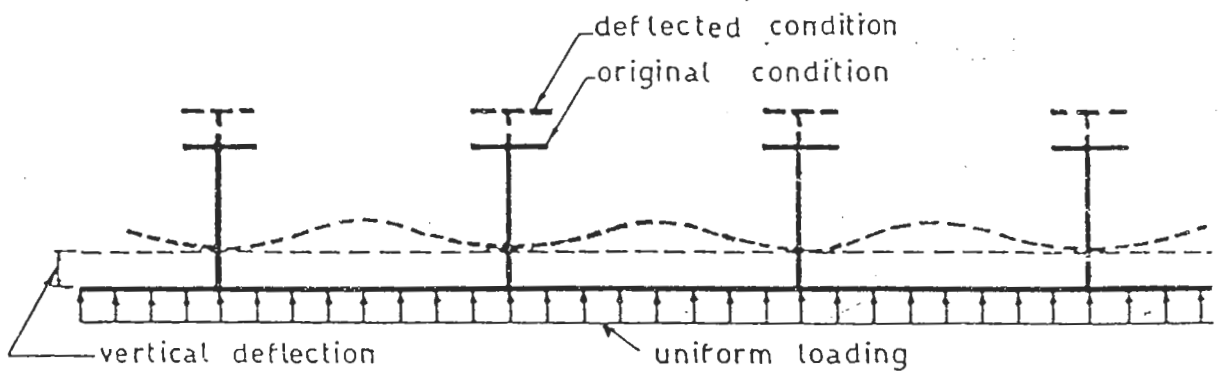


FIG. (4) LOADING ON LONGITUDINAL



(a) Asymmetrical Sections.



(b) Symmetrical Sections.

Fig.( 5 ), Mode of Deformation.

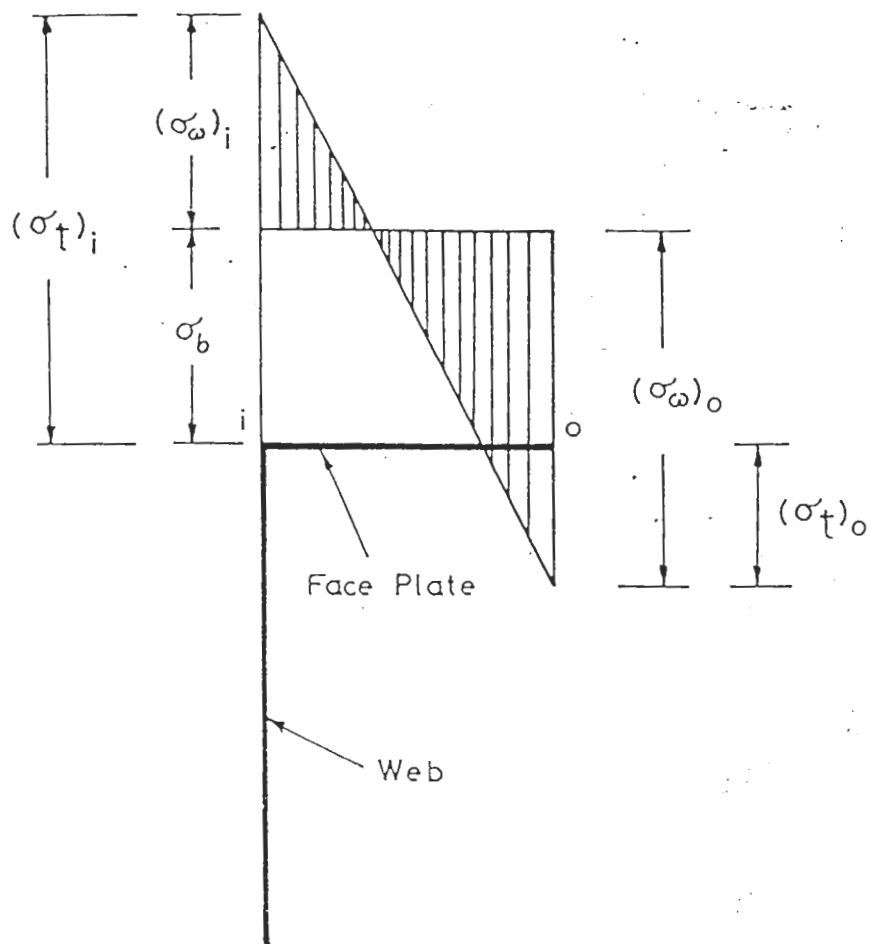


FIG. (6) FLEXURAL BENDING AND WARPING STRESSES

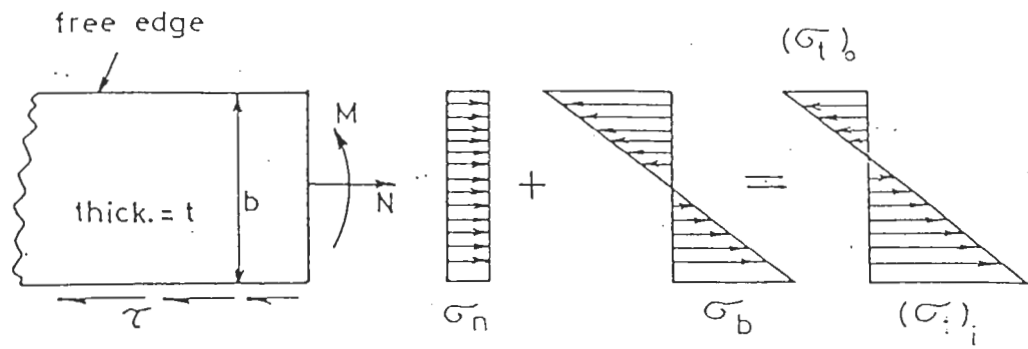


Fig. (7) Stresses due to shear loading

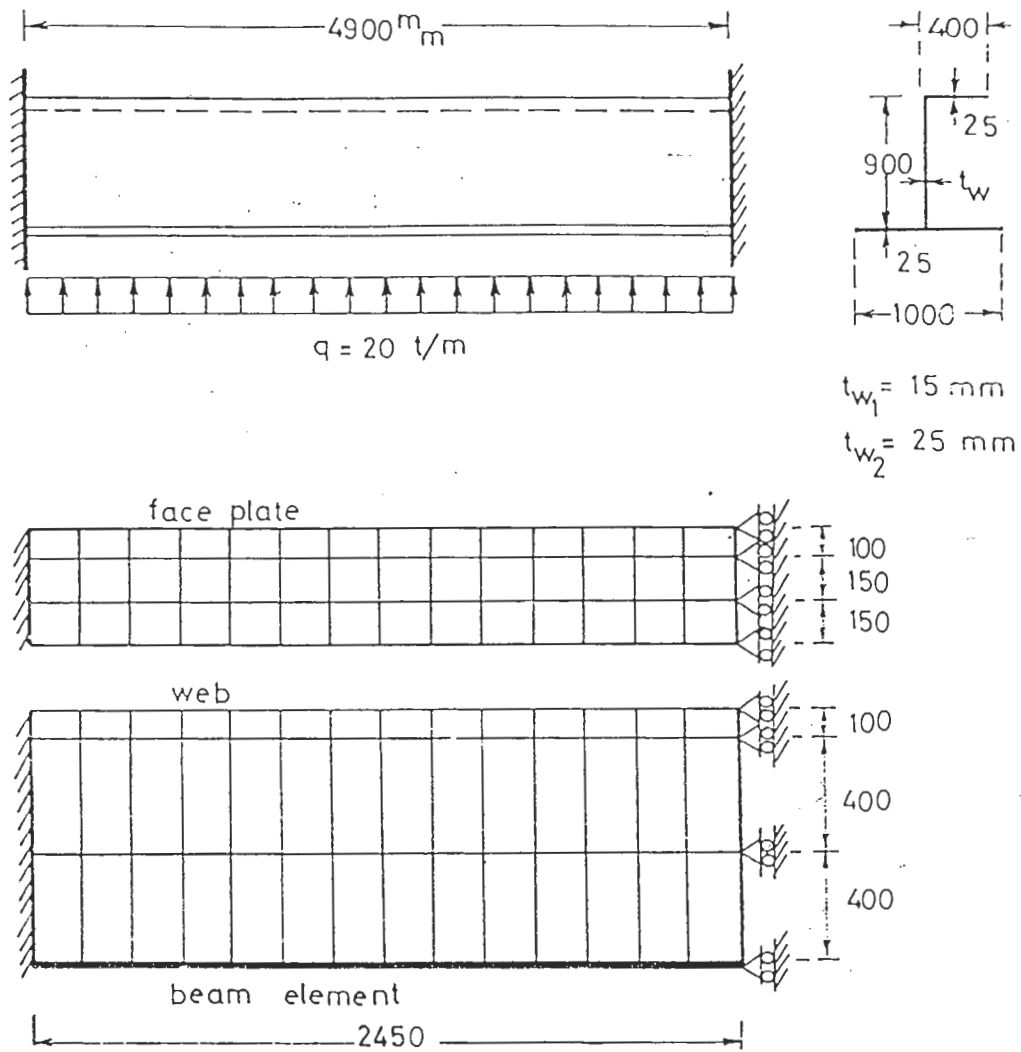


Fig.(8) Structure idealization

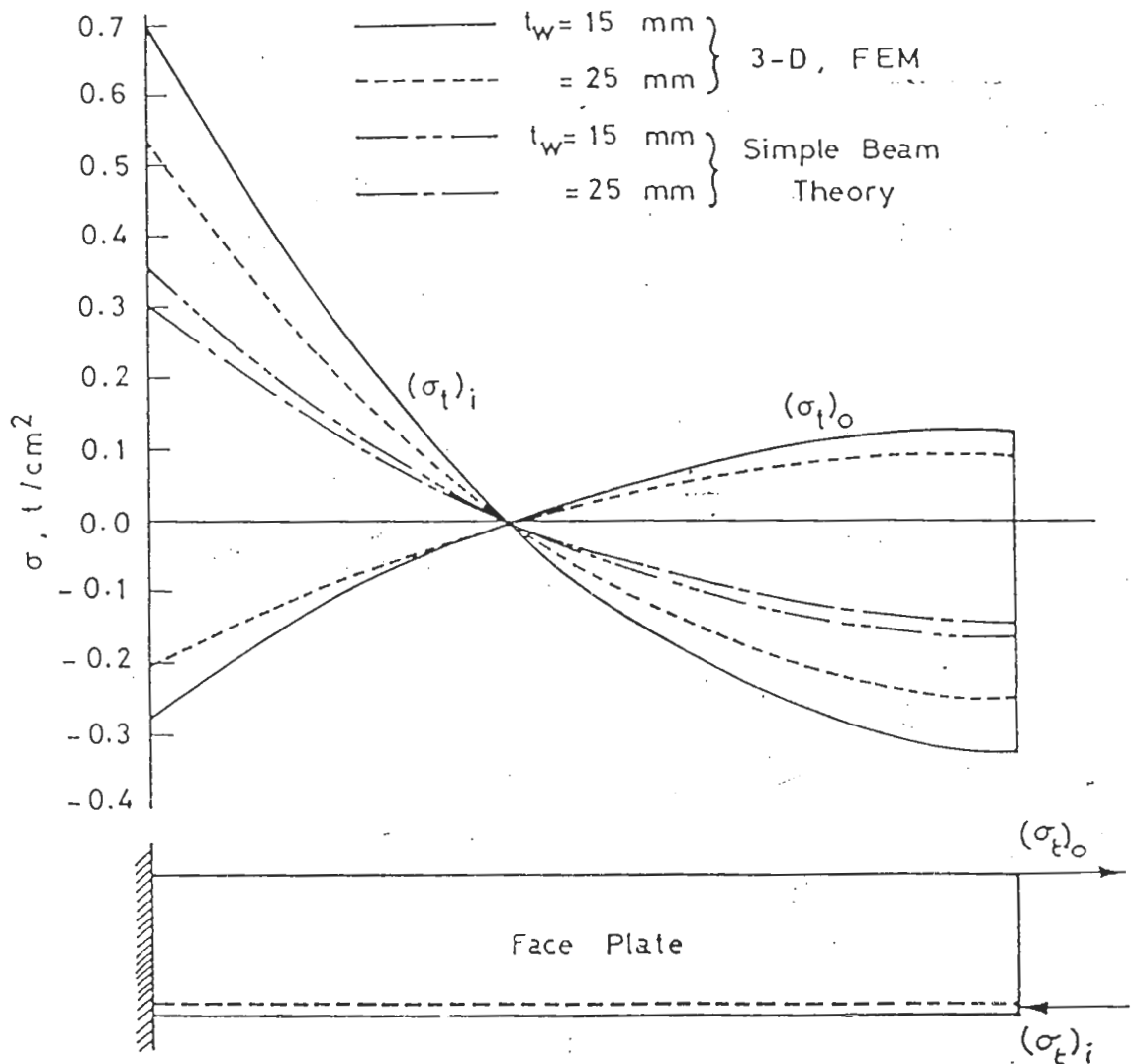


Fig. (9), DISTRIBUTION OF THE INNER AND OUTER STRESSES ALONG THE FACE PLATE



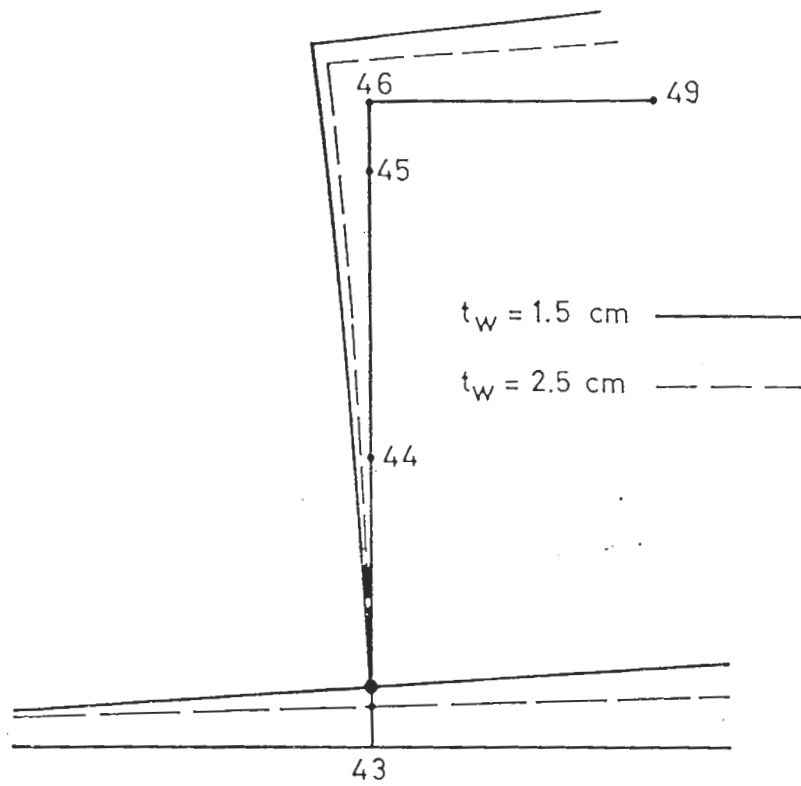
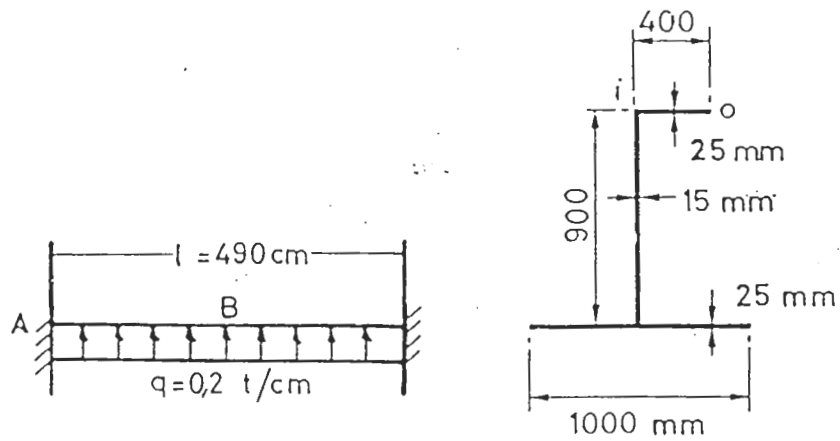


FIG. (10), DEFORMED SHAPE UNDER LATERAL LOADING



$A_b$  = sectional area of idealized bar.

$A_f$  = sectional area of face plate.

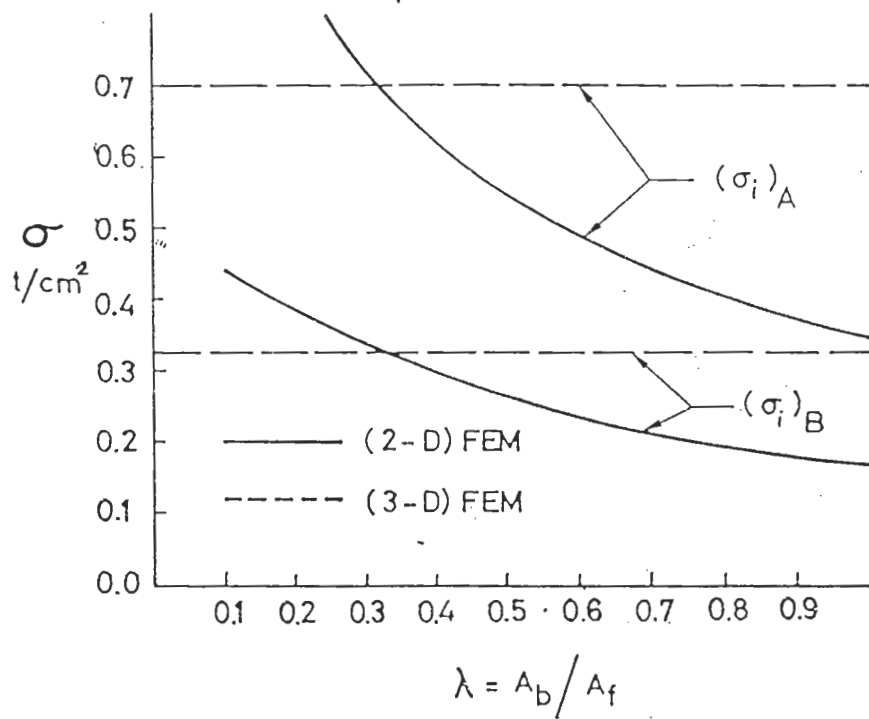


FIG.(11).EFFECTIVENESS OF FACE PLATE

SCALE  
0 1 2 mm

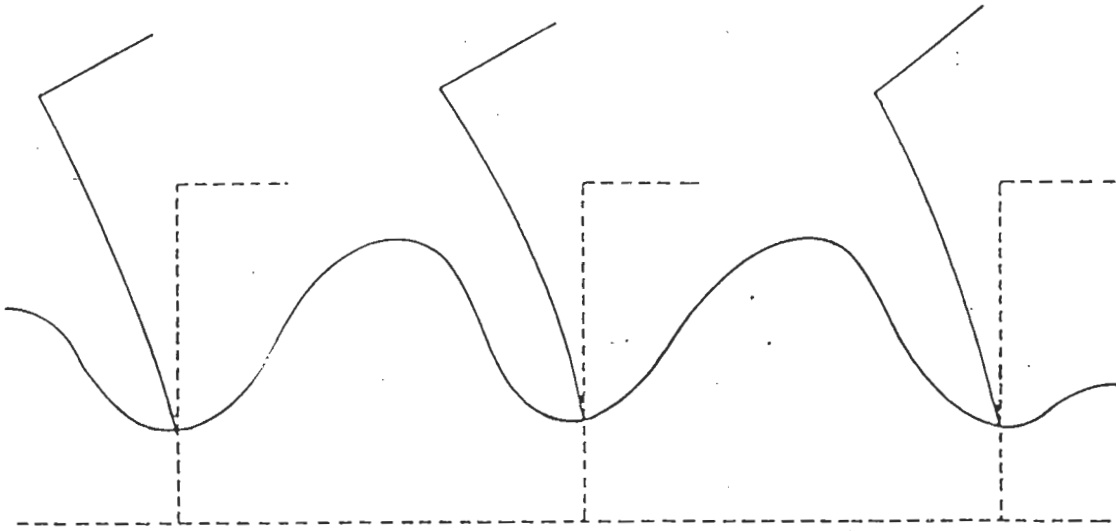
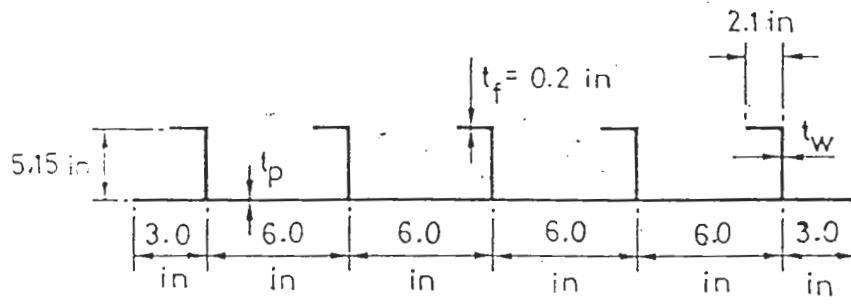


Fig.(12), DEFORMED SHAPE OF SECTION.



$$t_p = t_w = 0.101 \text{ in}$$

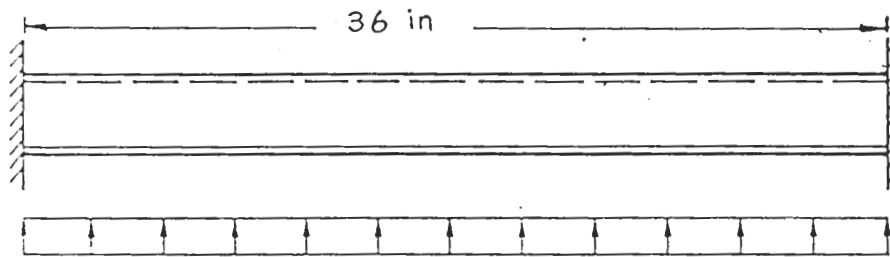


FIG.(14). TEST MODEL

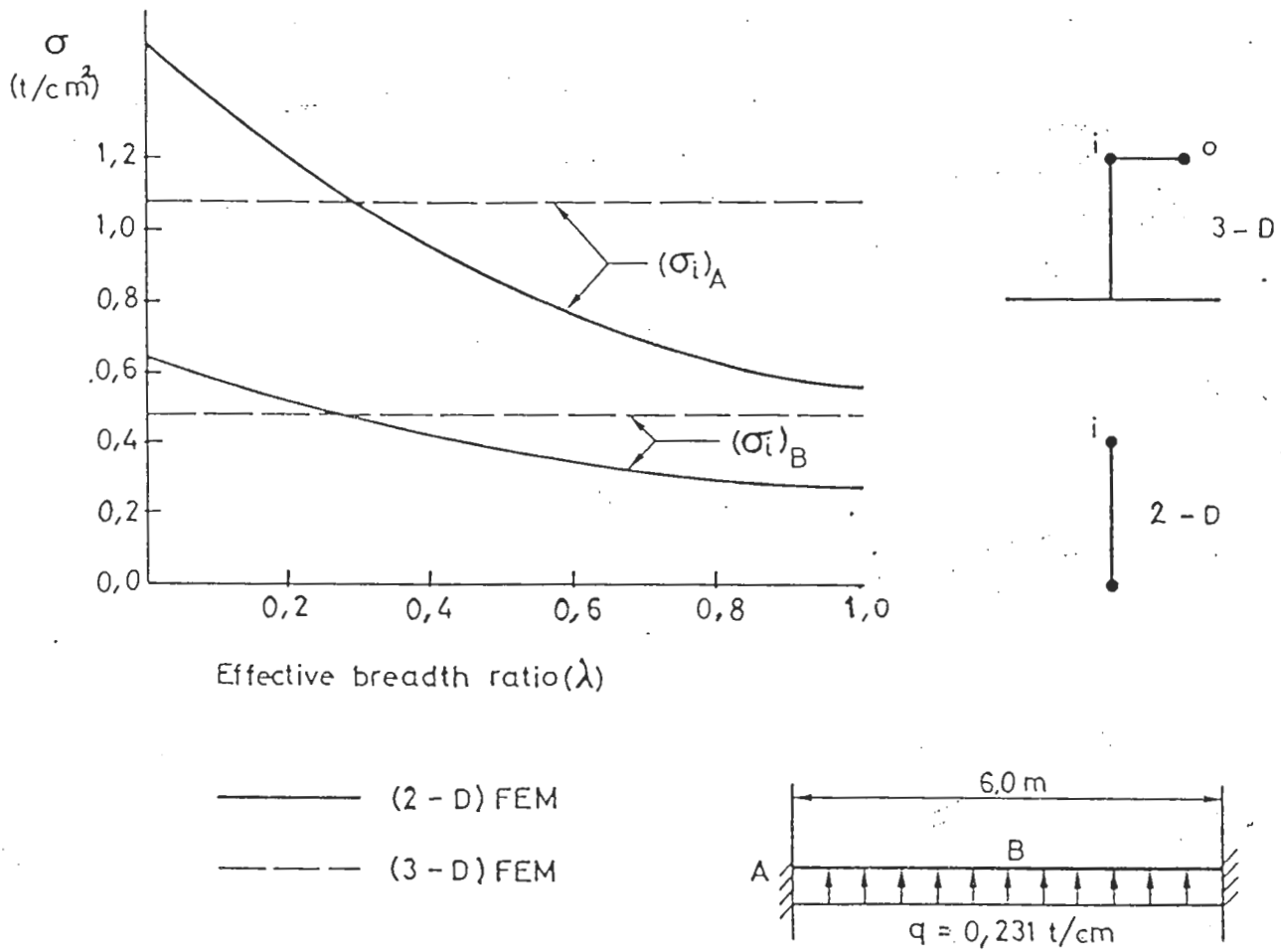
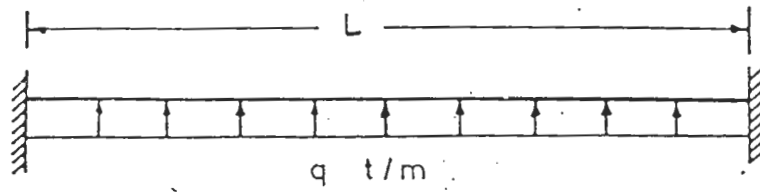


Fig.(13) CENTRAL AND END STRESSES FROM 2-D AND 3-D ANALYSIS

1) Uniform Loading



- |                         |                      |
|-------------------------|----------------------|
| a) $\phi = \phi' = 0$   | $\phi = \phi' = 0$   |
| b) $\phi = \phi'' = 0$  | $\phi = \phi'' = 0$  |
| c) $\phi = \phi''' = 0$ | $\phi = \phi''' = 0$ |

2) Triangular Loading

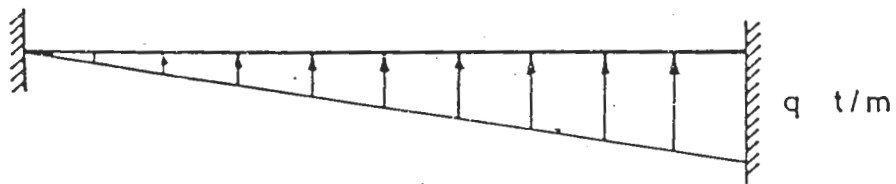


FIG. (15) CONDITIONS OF SUPPORT